Machine Learning for Quantum Computing

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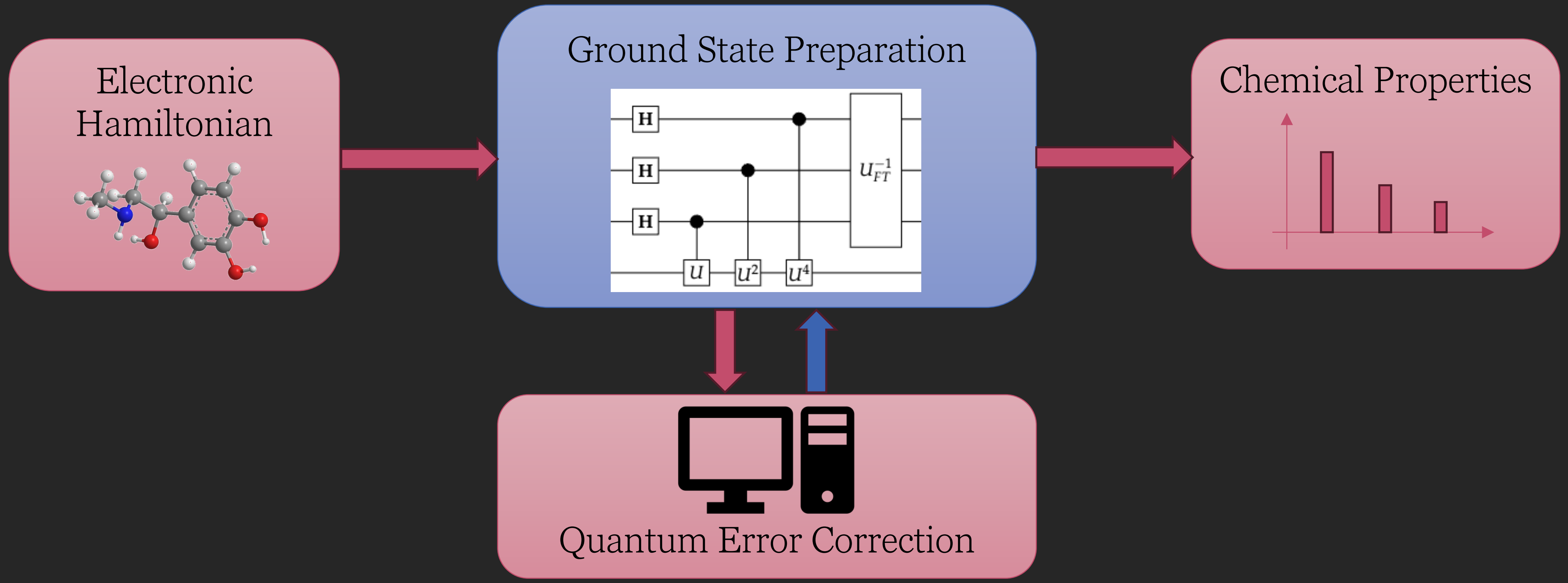
Outline

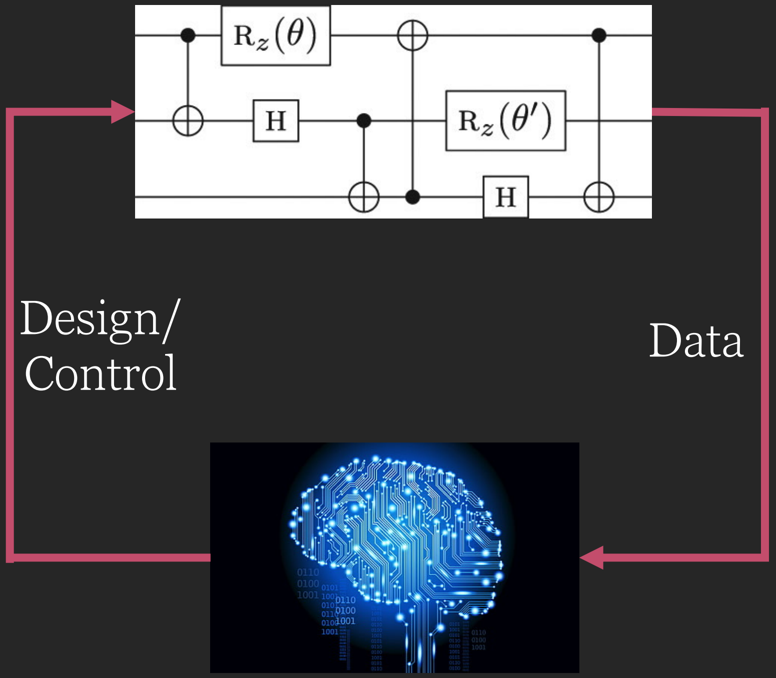
- Introduction: Machine Learning for Quantum Data
- Neural Decoders for Quantum Error Correction
- Exact Complexity Learning Long-Range Quantum Systems

Introduction

Quantum Computing Workflow



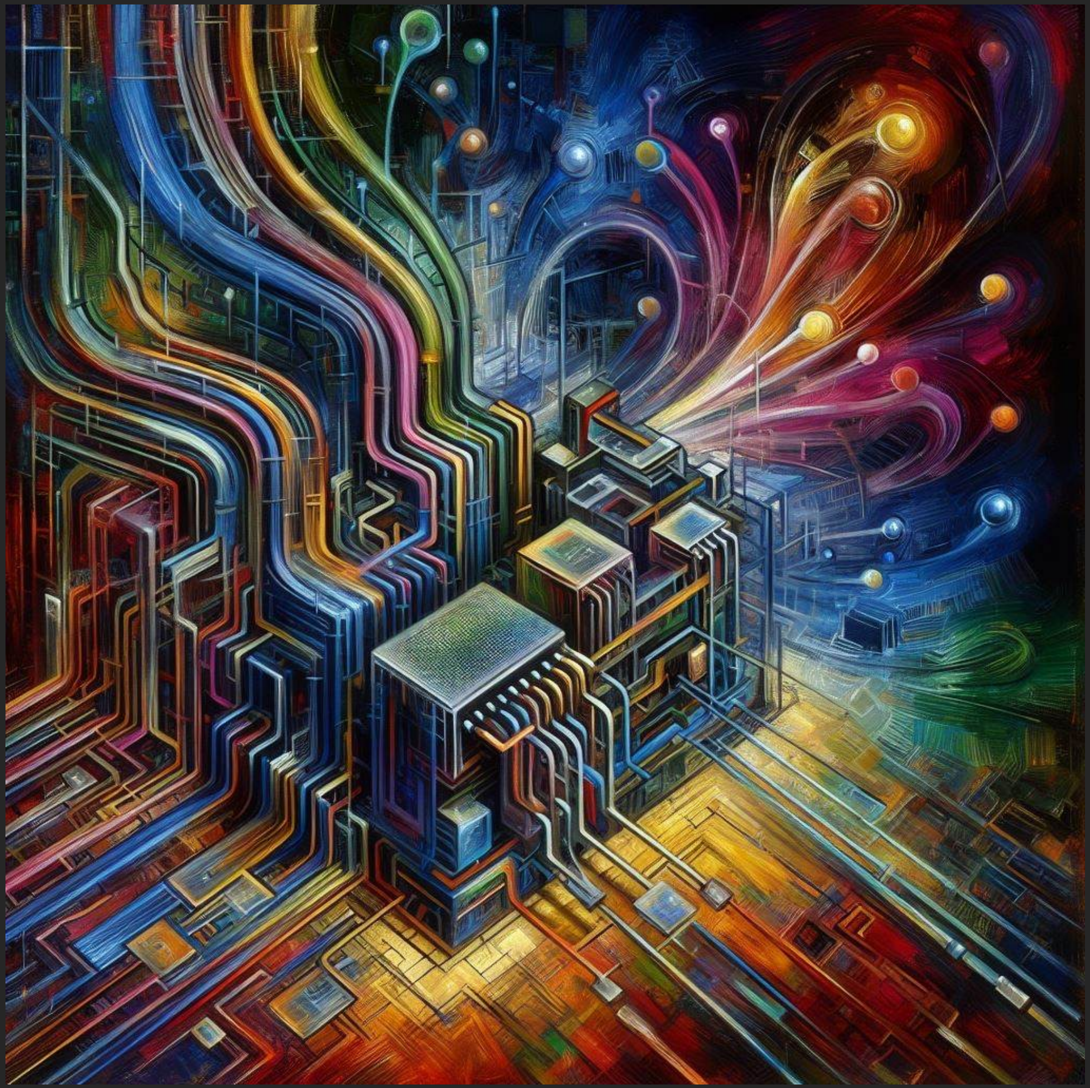
Learning from Quantum Data



Learning quantum states critical to control quantum resources

Hardware models & design quantum algorithms

Complexity theory



Prompt: “a painting of a quantum computer”

Machine Learning

- Ubiquitous image and text
- Approximate unknown function from input/output training data
- Challenges quantum data:
  1. *Data scarce*
  2. *Cannot learn arbitrary states*

Neural Decoders for Quantum Error Correction

- E EGOROV (UVA), RB, M WELLING (UVA)

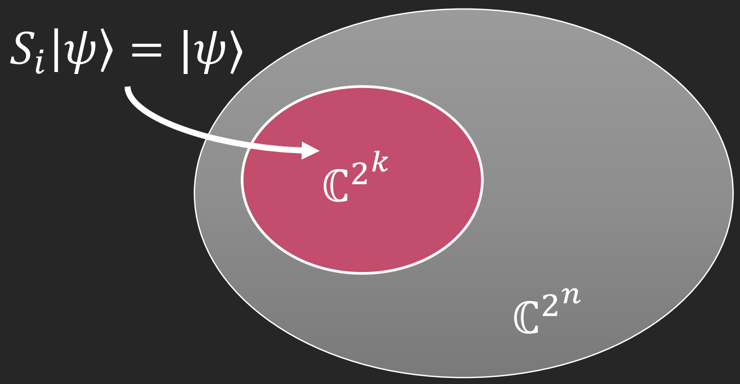
- HTTPS://ARXIV.ORG/ABS/2304.07362

Quantum Codes

- Stabilizer

$$\mathcal{S} = \langle S_1, \dots, S_\ell \rangle, \qquad [S_i, S_j] = 0$$

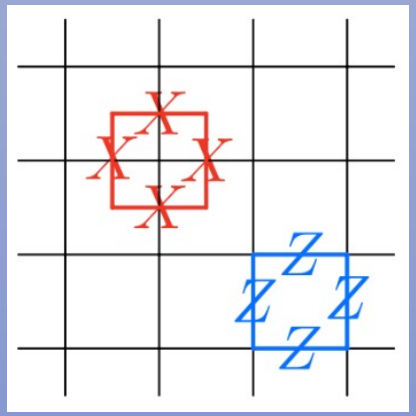
- Quantum code = $+1$ eigenspace of $\mathcal{S}$



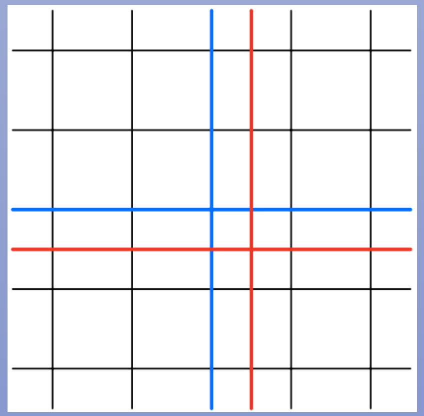
- Logical ops commute with $\mathcal{S}$ but indep.

Example: Toric Code ($k = 2$)

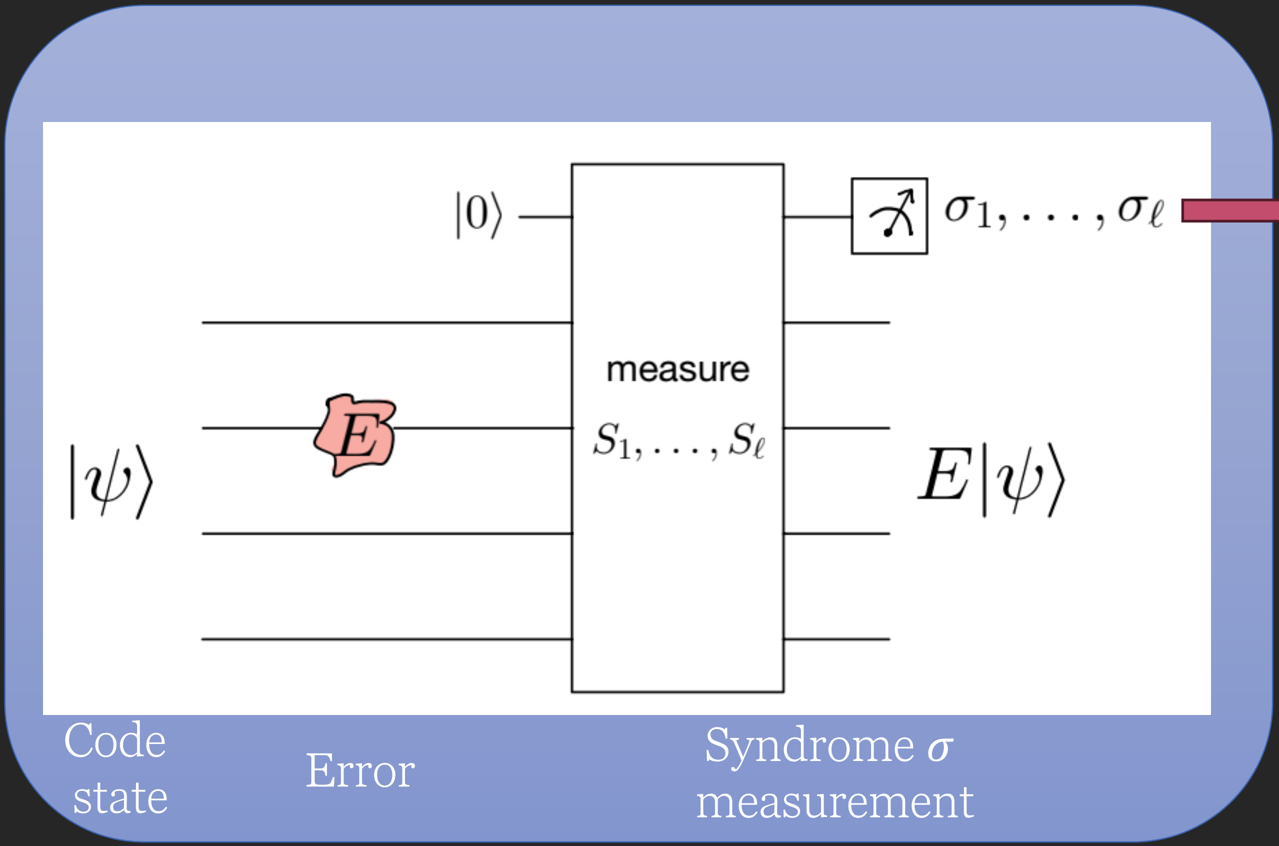
Stabilizer



Logical op

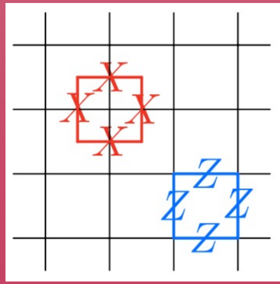


The Decoding Problem

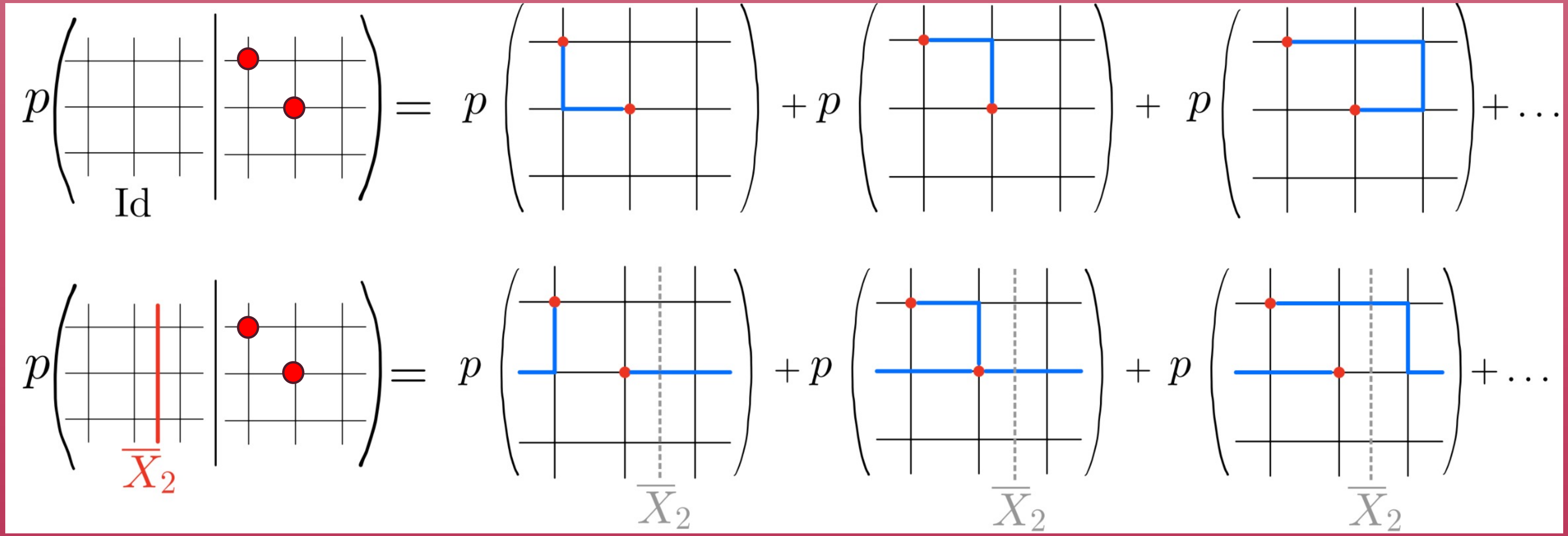


- $\sigma_i = 0$ if $ES_i = S_iE$
- $\sigma_i = 1$ if $ES_i = -S_iE$
- Logical errors are undetected

Optimal decoder: most likely logical error



$p(\cdot)$: noise model



Stat Mech Approach

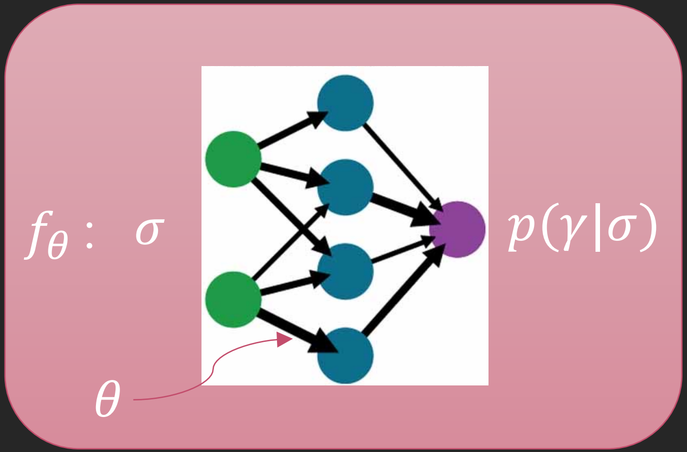
- Prob(Logical γ|Syndrome σ): sum compatible errors
- Partition function of disordered model
- Toric code bit flip is solvable! (Random bond Ising)
- In general, intractable… #P hard, so approximate

Machine Learning Approach

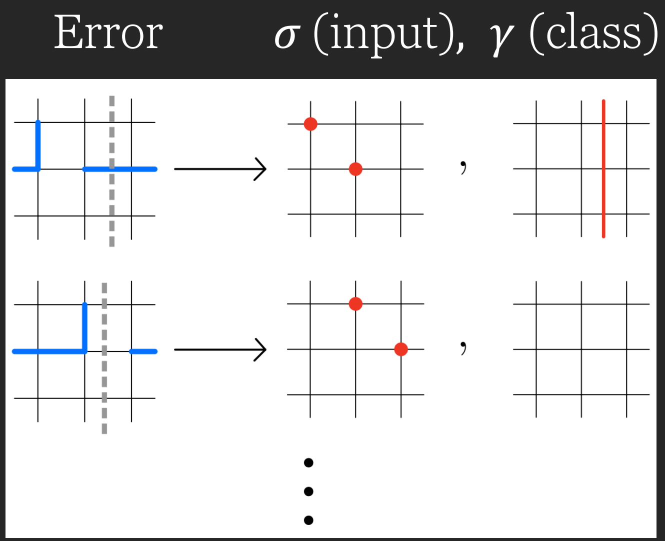
- $p(\gamma|\sigma)$ is solution to max likelihood:

$$\max_{f} \mathbb{E}(\log f(\gamma|\sigma))$$

- $\gamma, \sigma \sim$ prob. induced to noise model
- Variational approach: neural networks



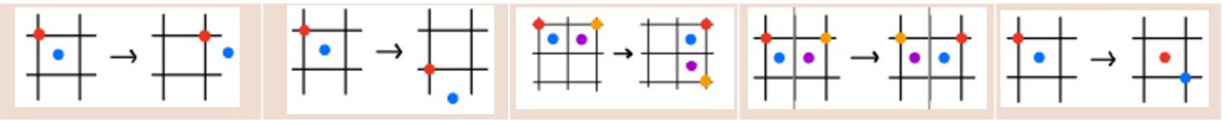
- Monte Carlo approximation (easy)

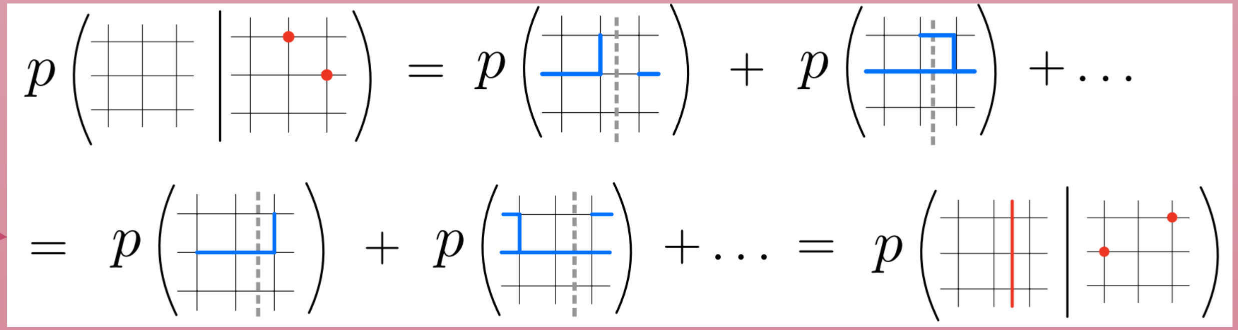


Automorphism Equivariance

Logical probabilities equivariant under automorphism group (qubit permutations that preserve the code space)

Horizontal translation	Vertical translation	Rotation **90°**	Horizontal flip	Duality





$p(\cdot)$ invariant

Results

- We construct neural networks equivariant under automorphism group of the code
- State-of-the-art compared to other neural decoders and popular heuristics
- $O(n^2)$ samples suffice to learn $p(\gamma|\sigma)$ for any noise*

*: [RB (to appear)]

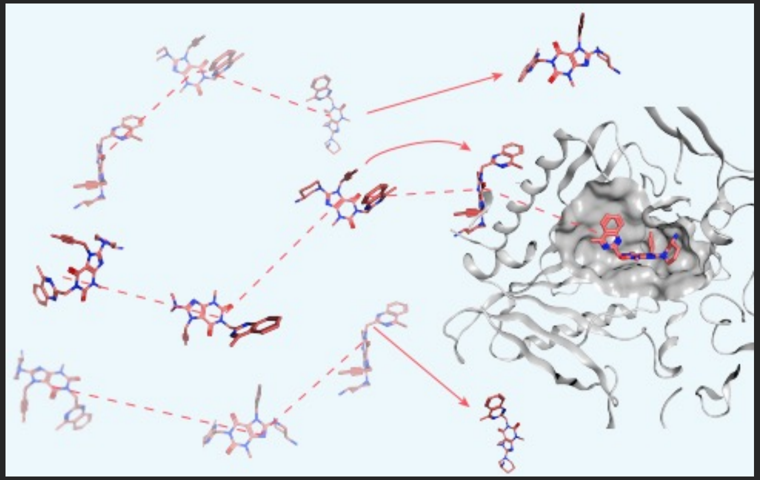
Learning Properties of Long-Range Quantum Systems

- Š ŠMÍD,(ICL), RB

- HTTPS://ARXIV.ORG/ABS/2312.17019

Problem Statement

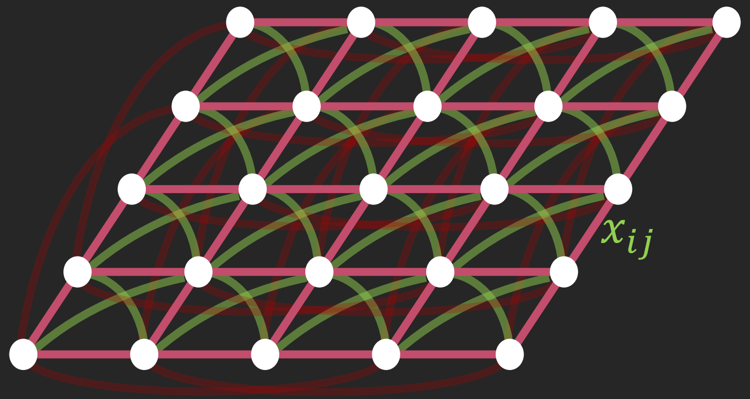
- Molecular dynamics: solve electronic problem for many nuclei positions



Billions times, each run ~ days
[Santagati et al, ‘23]

- Given $(x_1, \rho(x_1)), \ldots, (x_N, \rho(x_N))$, $\rho(x)$ g.s. $H(x) = \sum_{ij} h_{ij}(x_{ij})$.
- Find f small generalization error:

$$\mathbb{E}\left(f(x, O) - \mathrm{Tr}(\rho(x) O)\right)^2 < \epsilon$$



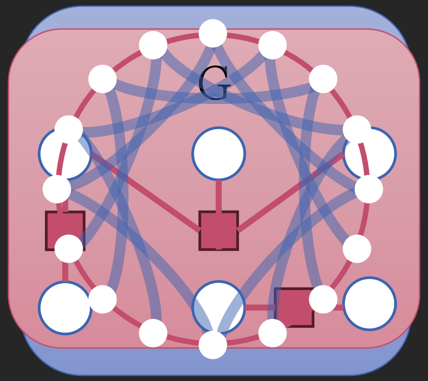
Analytical Results

Theorem: Let $H(x)$ be gapped. We can learn all $O = \sum_I O_I \, , O_I$ supported on k qubits with

$$N = \log(n) \begin{cases} 2^{polylog\left(\frac{1}{\epsilon}\right)} & exp.\, decay, short\ range^{*} \\ 2^{\epsilon^{-\omega} \log\left(\frac{1}{\epsilon}\right)} & power\ law\ \alpha > 2D. \end{cases}$$

Computing f takes $O(n^k N)$ time. Instead, computing $\mathrm{Tr}(O\rho(x))$ NP-hard (3SAT).

Corollary (Equivariance reduction)
$N = \log\left(n^k / |Aut(G)|\right)$ with G interaction hypergraph

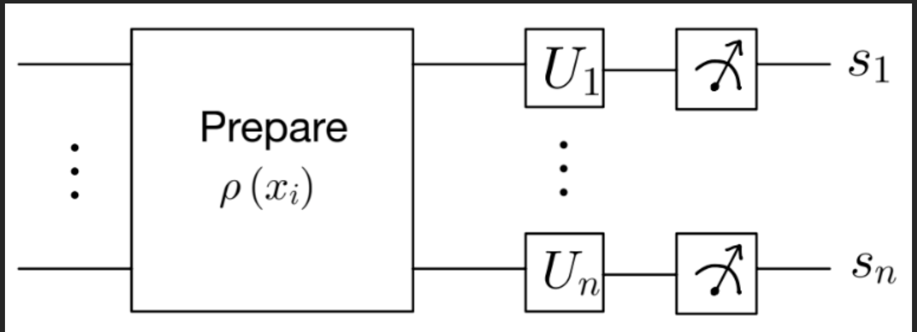


Corollary**: sum of k-local observables for periodic boundaries can be learned from a single sample

* [Lewis et al, '24. Onorati et al. '24] **[S Smid, RB (to appear)]

Proof I: Classical Dataset

- Compute classical shadows* for each x_i:

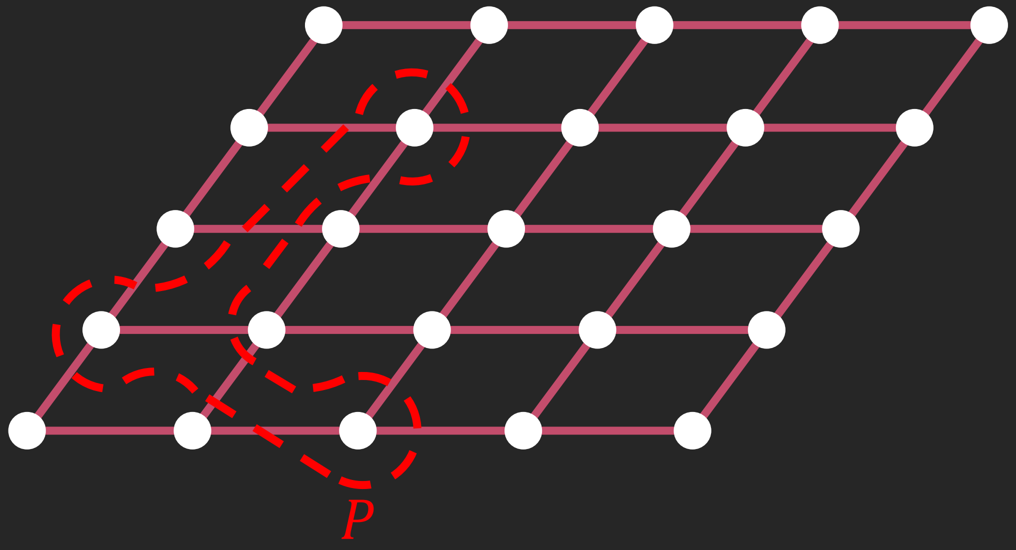


- *All* k-body red. density mat. $T = O(\log(n))$

$$\sigma_T(\rho) = \frac{1}{T}\sum_{t=1}^{T}\bigotimes_{i=1}^{k}\left(3\,|s_i^t\rangle\langle s_i^t| - 1\right)$$

- Classical data for Paulis weight k:

$$\{x_i, \mathrm{Tr}(\sigma_T(\rho(x_i))\,P) \approx \mathrm{Tr}(\rho(x_i)\,P)\}_{i=1}^{N}$$



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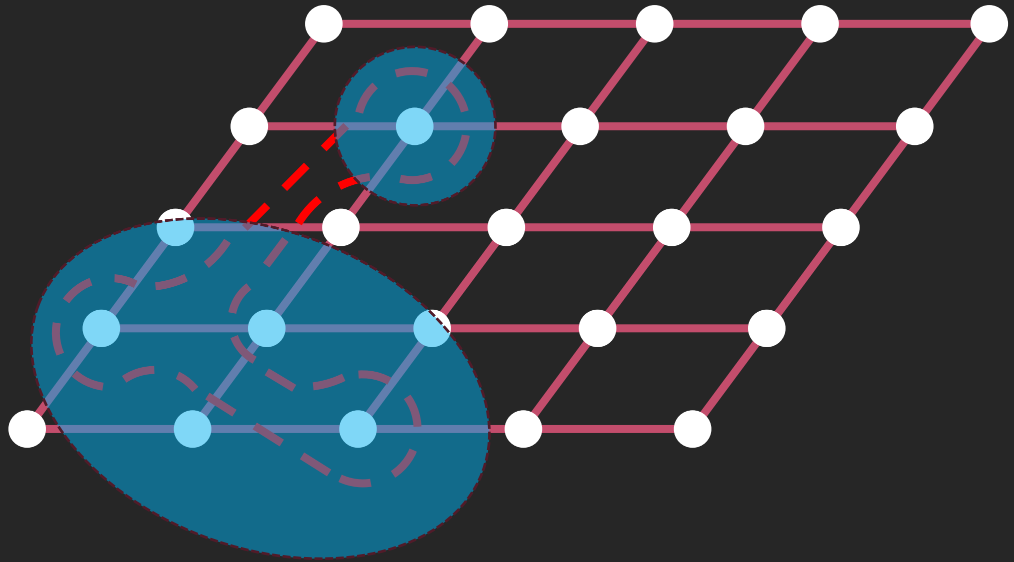
* Huang et al, '20

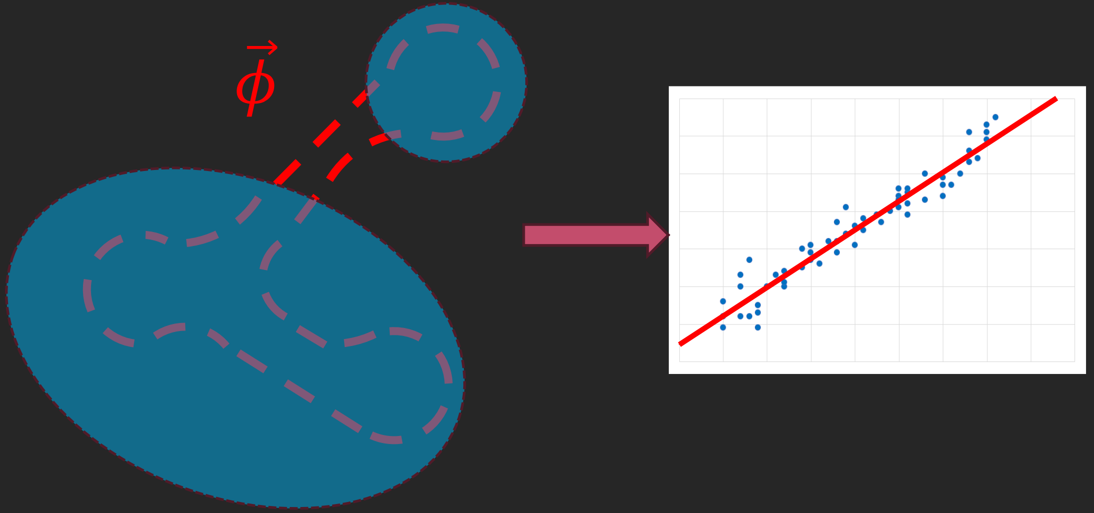
Proof II: Local Approximation

$$\mathrm{Tr}\big(P\rho(x)\big) \approx \mathrm{Tr}\left(P\rho\left(x\big|_S\right)\right)$$



- $\mathrm{Tr}\big(P\rho(x|_S)\big)$: $O(1)$ vars $\rightarrow M = O(1)$

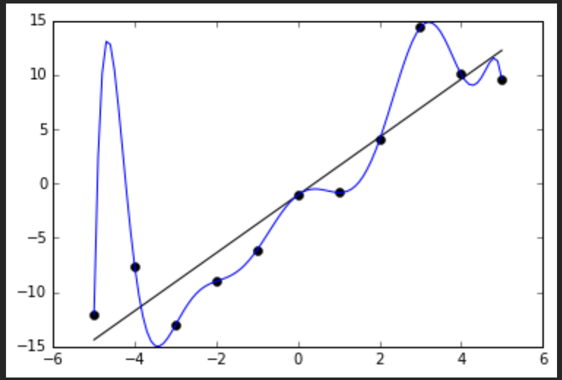
$$\mathrm{Tr}\left(P\rho\left(x\big|_S\right)\right) \approx \sum_{j=1}^{M} w_j \phi_j\left(x\big|_S\right)$$



- Use spectral flow + Lieb-Robinson bound*

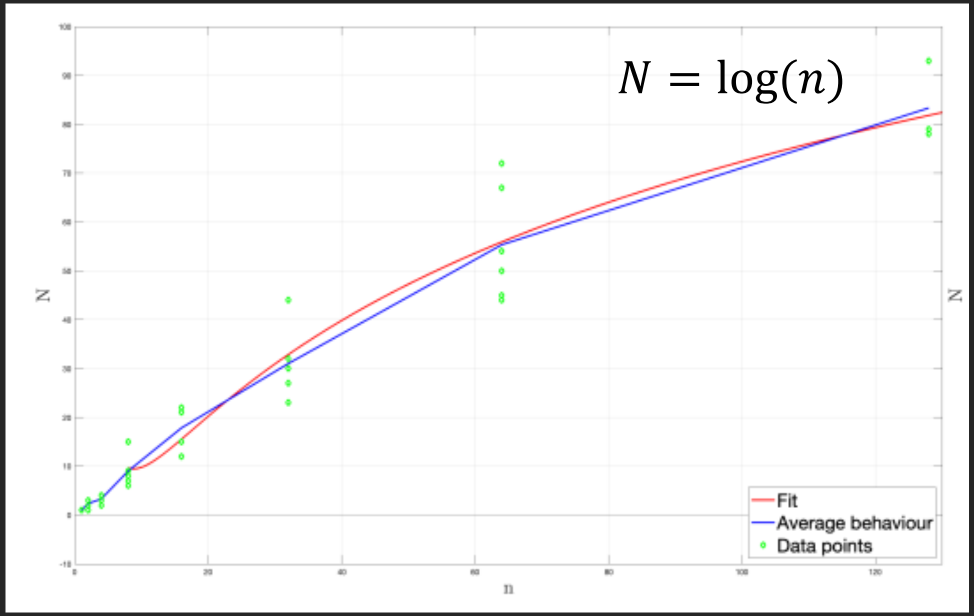
* Tran et al, '21

Further Results



- "Overfitting": if $\mathbb{P}(f(x) \neq y) > \epsilon$,
  $\mathbb{P}(f(x_1) = y_1, \dots, f(x_N) = y_N) \leq (1-\epsilon)^N$
- Prob $\exists$ a Pauli weight k st f^P overfits:
  $n^k(1-\epsilon)^N = O(1) \rightarrow N = O(\log(n))$

- DRMG Long range random Ising $\alpha = 3$



Conclusion

Learning quantum states critical for realizing quantum computers

Novel neural decoders for quantum error correction that achieve SOTA

Performance guarantees, extension realistic noise & LDPC codes

Real time decoding

Exact complexity learn gapped phases power law interactions

Poly error dep, Coulomb interactions

Quantum chemistry applications

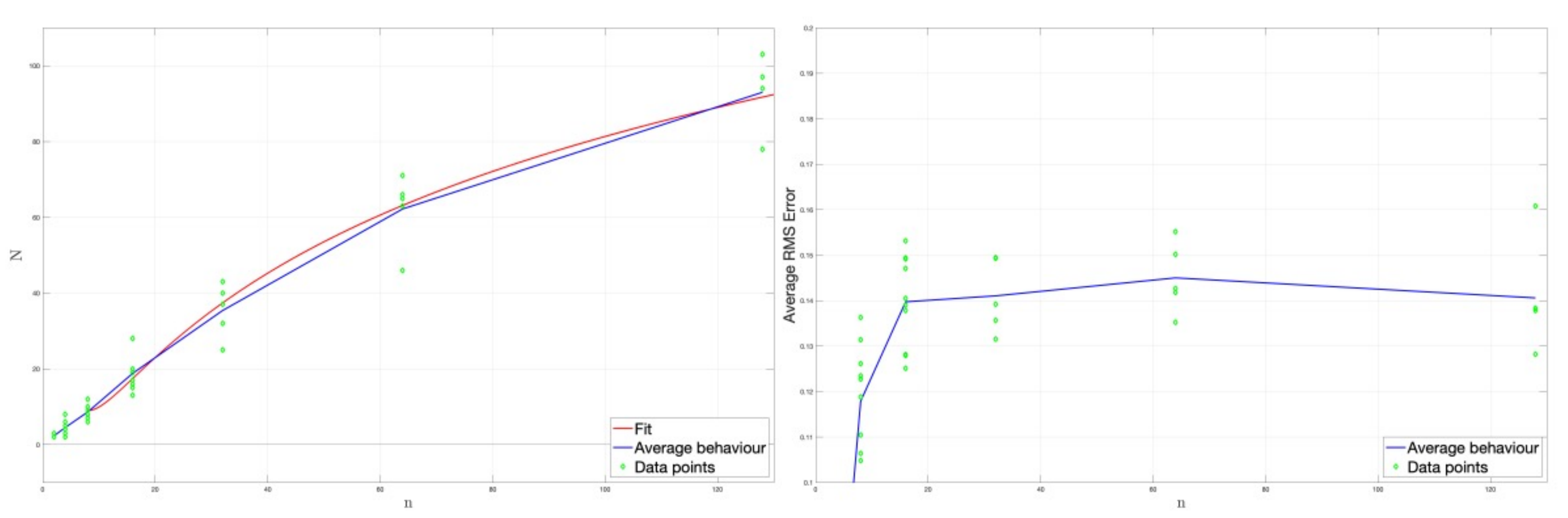
Efficient tomography of Bethe wavefunctions?

MPS, Stabilizer, Free fermions efficient

Backup Slides

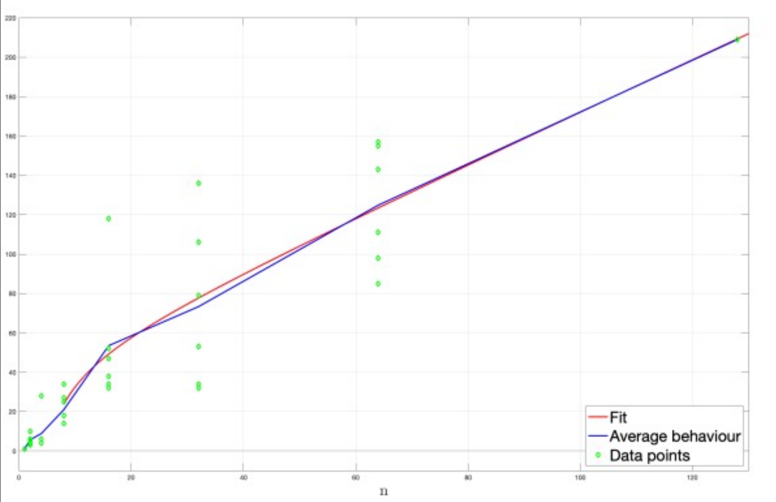
More Plots Learning Quantum Systems

Disordered Heisenberg nearest neighbors



(a) Open boundary conditions (b) Periodic boundary conditions

Long range Ising $\alpha = 1.5$



(b) $\alpha = 1.5$